I B.Tech Regular Examinations,June 2010
MATHEMATICS-1
Common to ME, CHEM, BME, IT, MECT, MEP, AE, BT, AME, ICE,
E.COMP.E, MMT, ETM, EIE, CSE, ECE, EEE,CE
Time: 3 hours   Max Marks: 75
Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Find the radius of curvature at any point on $y^2 = 4ax$ and hence show that the radius of curvature at the vertex is equal to the semi latus rectum.

   (b) Trace the curve $r = a\left(1 + \cos\theta\right)$ [7+8]

2. (a) Find the volume of Spherical cap of height h cut off from a sphere of radius a.

   (b) Evaluate $\int\limits_{0}^{\pi}\int\limits_{0}^{a(1+\cos\theta)} r^2 Cos\theta \, dr \, d\theta$ [8+7]

3. (a) Solve the differential equation $(D^2 + D + 1)y = x^3$

   (b) Solve the differential equation $(D^2 + 1)y = \sin x \sin 2x$ [8+7]

4. (a) Form the differential equation by eliminating arbitrary constants
      y = a x^3+bx^2

   (b) Solve the differential equation $x^3 \frac{dy}{dx} = y^3 + y^2\sqrt{y^2 - x^2}$

   (c) Find the orthogonal Trajectories of the family of curves x^2+y^2 = a^2 [4+6+5]

5. (a) If $u = x^2 - 2y, v = x + y + z, w = x - 2y + 3z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

   (b) Find the maximum and minimum values of $f(x) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$
                                                                          [8+7]

6. (a) Find the constants a and b so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (-1,1,2).

   (b) Evaluate $\oint_e (yzdx + xzdy + xydz)$ over arc of a helix $x = a\cos t, y = a\sin t, z = kt \; as \; t$ varies from 0 to 2π [8+7]

7. (a) Find $L\left[\frac{e^{-t}\sin t}{t}\right]$

   (b) Solve the following differential equation using the Laplace transforms
      $\frac{d^2 x}{dt^2} - 2\frac{dx}{dt} + x = e^{2t}$ with $x(0) = 2, \; \frac{dx}{dt} = -1 \; at \; t = 0$ [8+7]

8. (a) Test the convergence of the series $\frac{(n!)^2 x^{2n}}{(2n)!}$

   (b) Test the convergence of the series $\sum \frac{\left(\sqrt{5}-1\right)^n}{n^2+1}$ [7+8]

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